MATH 141 HOMEWORK 10 DUE DATE: THURSDAY, JUNE 22ND

Name: Solutions

This homework is due Thursday, June 22nd during recitation. If you have questions regarding any of this, feel free to ask during office hours or send me an email. When writing solutions, present your answers clearly and neatly, showing only necessary work.

1. Use the ε-δ definition of a limit to find the following:

(a) $\lim_{x\to 2} 3x+5$

$|f(x)-L|<\varepsilon$

$|3x+5-11|<\varepsilon$

$|3x-6|<\varepsilon$

$3|x-2|<\varepsilon$

$|x-2|<\varepsilon/3$

$0<|x-c|<\delta$

$0<|x-2|<\delta$

$\boxed{\text{let } \delta=\varepsilon/3}$

Answer: $\lim_{x\to 2} f(x) = 11$

(b) $\lim_{x\to 4} x^2+3x+1$

$|f(x)-L|<\varepsilon$

$|x^2+3x+1-29|<\varepsilon$

$|x^2+3x-28|<\varepsilon$

$|(x+7)(x-4)|<\varepsilon$

$|x+7||x-4|<\varepsilon$

$|x-4|<\varepsilon/|x+7|$

$0<|x-c|<\delta$

$0<|x-4|<\delta$

We can assume $|x-4|<1$

$\Rightarrow -1<x-4<1$

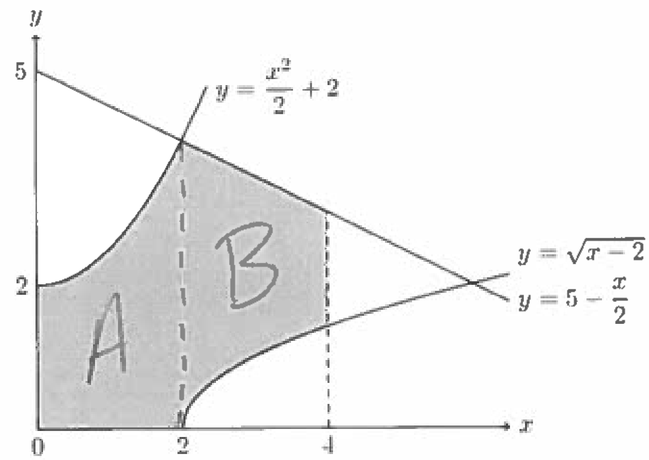
$\Rightarrow 10<x+7<12$

$\Rightarrow |x+7|=x+7<12$

$\Rightarrow$ let $\boxed{\delta=\varepsilon/12}$

Answer: $\lim_{x\to 4} f(x) = 29$

2. Find the area of the shaded region below;



$$A = \int_0^2 \frac{1}{2}x^2 + 2\,dx = \frac{1}{6}x^3 + 2x \Big|_0^2$$

$$= \frac{1}{6}(2)^3 + 2(2) - \frac{1}{6}(0)^3 + 2(0) = \frac{8}{6} + 4 = \frac{4}{3} + \frac{12}{3} = \frac{16}{3}$$

$$B = \int_2^4 5 - \frac{1}{2}x\,dx - \int_2^4 \sqrt{x-2}\,dx$$

$$= 5x - \frac{1}{4}x^2 \Big|_2^4 - \frac{2}{3}(x-2)^{3/2} \Big|_2^4$$

$$= 5(4) - \frac{1}{4}(4)^2 - 5(2) + \frac{1}{4}(2)^2 - \frac{2}{3}(4-2)^{3/2} + \frac{2}{3}(2-2)^{3/2}$$

$$= 20 - 4 - 10 + 1 - \frac{4}{3}\sqrt{2} + 0$$

$$= 7 - \frac{4}{3}\sqrt{2}$$

$$A + B = \frac{16}{3} + 7 - \frac{4}{3}\sqrt{2} = \frac{16}{3} + \frac{21}{3} - \frac{4\sqrt{2}}{3}$$

$$= \frac{37 - 4\sqrt{2}}{3}$$

Answer: $\frac{37 - 4\sqrt{2}}{3}$

3. Evaluate the following limits:

(a) $\lim_{x\to 4} \frac{3x-12}{x^2-16}$ $= \lim_{x\to 4} \frac{3(x-4)}{(x+4)(x-4)} = \lim_{x\to 4} \frac{3}{x+4} = \frac{3}{4+4}$

Answer: $3/8$

(b) $\lim_{x\to 0} \frac{\sin(6x)}{4x}$ $\overset{L'H}{=} \lim_{x\to 0} \frac{6\cos(6x)}{4} = \frac{6}{4}$

Answer: $3/2$

(c) $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x^2-9}$ $= \lim_{x\to 3} \frac{\frac{3-x}{3x}}{(x+3)(x-3)} = \lim_{x\to 3} \frac{-(x-3)}{3x(x+3)(x-3)}$

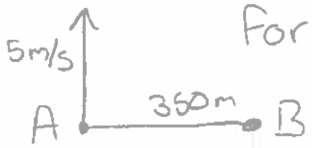
$= \lim_{x\to 3} \frac{-1}{3x(x+3)} = \frac{-1}{3(3)(3+3)}$

Answer: $-1/54$

(d) $\lim_{x\to 5} \frac{\sqrt{25-x^2}-5}{x}$ $= \frac{\sqrt{25-5^2}-5}{5} = \frac{-5}{5} = -1$

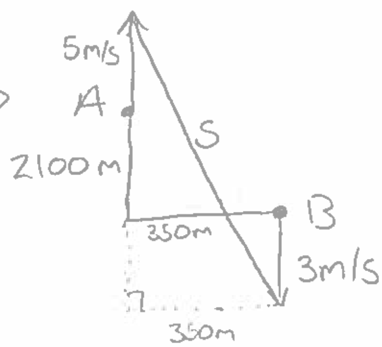
Answer: -1

4. Two people on bikes are 350m apart. Person A starts riding north at a rate of 5m/sec and 7 minutes later Person B starts riding south at 3m/sec. At what rate is the distance separating the two people changing 25 minutes after Person A starts riding?



For 7mins = 420secs

Distance A travels:
$2100 + 5t$
Distance B travels:
$3t$



$$c^2 = (2100+8t)^2 + 350^2$$

$$t = 25 - 7\text{mins} = 1080\text{secs}$$

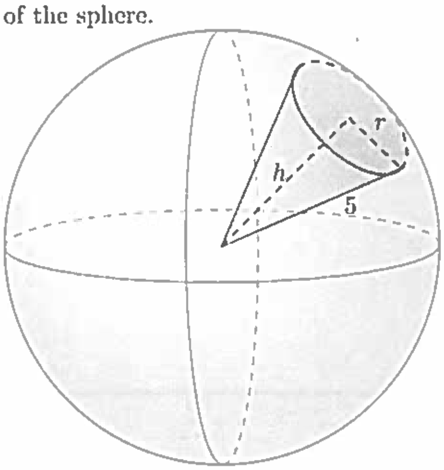
$$c = \sqrt{(10740)^2 + 350^2}$$

$$2c\frac{dc}{dt} = 16(2100+8t)$$

$$\frac{dc}{dt} = \frac{8(2100+8t)}{c} = \frac{8(10740)}{\sqrt{(10740)^2+350^2}} \text{ m/s}$$

Answer: $\frac{8(10740)}{\sqrt{(10740)^2+350^2}}$ m/s.

5. Find the maximal volume of the cone that can be inscribed inside a sphere of radius 5cm, whose top point is the centre of the sphere.



$$r^2 + h^2 = 5^2 \Rightarrow r^2 = 25 - h^2$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(25-h^2)h$$

$$= \frac{\pi}{3}(25h - h^3)$$

$$\frac{dV}{dh} = \frac{\pi}{3}(25 - 3h^2) = 0 \Rightarrow h = \sqrt{\frac{25}{3}} = \frac{5\sqrt{3}}{3}$$

$$V = \frac{\pi}{3}\left(25\cdot\frac{5\sqrt{3}}{3} - \frac{25\cdot 3}{9}\cdot\frac{5\sqrt{3}}{3}\right)$$

$$= \frac{50\pi}{9}\cdot\frac{5\sqrt{3}}{3} = \frac{250\sqrt{3}\pi}{27} \text{ cm}^3$$

Answer: $\frac{250\sqrt{3}\pi}{27}$ cm^3

The End.